MATH 579: Combinatorics Exam 3

Please read the following instructions. For the following exam you may not use any papers, books, or computers. You may use a calculator. Please turn in **exactly four** problems. You must do problems 1-3, and one more chosen from 4-6. Number 7 is optional. Please write your answers on separate paper, make clear what work goes with which problem, adequately justify all answers, simplify all numerical answers as best you can, and put your name or initials on every page. You have 50 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by $\frac{5}{2}$ for your exam score.

Turn in problems 1,2,3:

1. How many anagrams does $AAAABBBCCCCD$ have?
2. Let $n \in \mathbb{N}_0$. Prove that $2^n = \sum_{k=0}^{n} \binom{n}{k}$.
3. Prove the "Hexagon Identity":
   For all $k \in \mathbb{N}$, $\binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1} = \binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k}$.
   For one bonus point, explain why it's called the Hexagon identity.

Turn in exactly one more problem of your choice:
You should be able to solve these using known formulas. Induction proofs will lose one point.

4. Let $n \in \mathbb{N}_0$. Prove that $\frac{1}{n+1} = \sum_{k=0}^{n} \frac{(-1)^k}{k+1}\binom{n}{k}$.
5. Let $m, n \in \mathbb{N}_0$. Prove that $\binom{m+n+1}{n} = \sum_{k=0}^{n} \binom{m+k}{k}$.
6. Let $n \in \mathbb{N}_0$. Prove that $\binom{2n+1}{n+1} = \sum_{k=0}^{n} \binom{n}{k}\binom{n+1}{k}$.

You may also turn in the following (optional):

7. Describe your preferences for your next group assignment. (will be kept confidential)

Please keep this sheet for your records.